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DISCUSSION OF
GROUND-WATER MOVEMENT
CONTROLLED THROUGH
SPREADING

(Published in August, 1951)

By David K. Todd, Max Suter, D. P. Krynine,
and Paul Baumann

IRRIGATION DIVISION

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<i>Technical Division</i>	<i>Proceedings-Separate Number</i>
Air Transport	42, 43, 48, 52, 60, 93, 94, 95, 100, 103, 104, 108, 121, 130 (Discussion: D-XXVIII, D-7, D-16, D-18, D-23, D-43, D-75)
City Planning	58, 60, 62, 64, 93, 94, 99, 101, 104, 105, 115, 131, 138 (Discussion: D-16, D-23, D-43, D-60, D-62, D-65, D-86)
Construction	43, 50, 55, 71, 92, 94, 103, 108, 109, 113, 117, 121, 126, 130, 132, 133, 136, 137 (Discussion: D-3, D-8, D-17, D-23, D-36, D-40, D-71, D-75, D-92)
Engineering Mechanics	122, 124, 125, 126, 127, 128, 129, 134, 135, 136, 139, 141 (Discussion: D-24, D-33, D-34, D-49, D-54, D-61, D-96, D-100)
Highway	43, 44, 48, 58, 70, 100, 105, 108, 113, 120, 121, 130, 137, 138 (Discussion: D-XXVIII, D-23, D-60, D-75)
Hydraulics	50, 55, 56, 57, 70, 71, 78, 79, 80, 83, 86, 92, 96, 106, 107, 110, 111, 112, 113, 116, 120, 123, 130, 134, 135, 139, 141 (Discussion: D-70, D-71, D-76, D-78, D-79, D-86, D-92, D-96)
Irrigation and Drainage	97, 98, 99, 102, 106, 109, 110, 111, 112, 114, 117, 118, 120, 129, 130, 133, 134, 135, 138, 139, 140, 141 (Discussion: D-XXIII, D-3, D-7, D-11, D-17, D-19, D-25-K, D-29, D-30, D-38, D-40, D-44, D-47, D-57, D-70, D-71, D-76, D-78, D-80, D-86, D-87, D-92, D-96)
Power	48, 55, 56, 69, 71, 88, 96, 103, 106, 109, 110, 117, 118, 120, 129, 130, 133, 134, 135, 139, 141 (Discussion: D-XXIII, D-2, D-3, D-7, D-38, D-40, D-44, D-70, D-71, D-76, D-78, D-79, D-86, D-92, D-96)
Sanitary Engineering	55, 56, 87, 91, 96, 106, 111, 118, 130, 133, 134, 135, 139, 141 (Discussion: D-29, D-37, D-56, D-60, D-70, D-76, D-79, D-80, D-84, D-86, D-87, D-92, D-96)
Soil Mechanics and Foundations	43, 44, 48, 94, 102, 103, 106, 108, 109, 115, 130 (Discussion: D-4, D-XXVIII, D-7, D-43, D-44, D-56, D-75 D-86)
Structural	42, 49, 51, 53, 54, 59, 61, 66, 89, 100, 103, 109, 113, 116, 117, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137 (Discussion: D-51, D-53, D-54, D-59, D-61, D-66, D-72, D-100)
Surveying and Mapping	50, 52, 55, 60, 63, 65, 68, 121, 138 (Discussion: D-60, D-65)
Waterways	41, 44, 45, 50, 56, 57, 70, 71, 96, 107, 112, 113, 115, 120, 123, 130, 135 (Discussion: D-8, D-9, D-19, D-27, D-28, D-56, D-70, D-71, D-78, D-79, D-80)

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DISCUSSION

DAVID K. TODD¹⁸ J. M. ASCE.—In connection with the application of spreading to control sea-water intrusion, the author states: (paragraph following Eqs. 17) "To maintain control indefinitely the rate of infiltration must be such as to cause rise of the mound to at least mean sea level." For an unconfined homogeneous isotropic aquifer the mound elevation must exceed that of mean sea level to develop an effective equilibrium condition in repelling sea-water intrusion. According to the Ghyben-Herzberg principle,^{19,20} the greater specific gravity of salt water will balance a correspondingly greater head of fresh water in saturated ground-water zones. Following the author's notation, this may be expressed as

$$\alpha_F (a_o + H_d + \Delta H) = \alpha_s (a_o + H_d) \dots \dots \dots (125)$$

in which α_F equals the specific gravity of fresh water; α_s the specific gravity of salt water; and ΔH , the incremental head of fresh water required on the mound to maintain equilibrium. Solving for ΔH ,

$$\Delta H = \frac{\alpha_s - \alpha_F}{\alpha_F} (a_o + H_d) \dots \dots \dots (126)$$

To illustrate the magnitude of the increased fresh-water head required, the author's typical prototype values of $a_o = 100$ ft and $H_d = 35$ ft may be inserted in Eq. 126, together with values²¹ of $\alpha_s = 1.026$ and $\alpha_F = 1.000$. A value of $\Delta H = 3.5$ ft is found. Thus, assuming that undiluted sea water is in direct contact with the fresh ground water, the fresh-water mound must be maintained more than 3 ft above mean sea level to control sea-water intrusion.

The author has limited his analytical treatment and model studies to cases of unconfined ground-water conditions only, and has not investigated the problem of pressure aquifers. However, the impression is given that an important application of spreading is that of control of sea-water intrusion. This application is important in coastal areas where unconfined aquifers do exist; but in California where the problem of a sea-water intrusion is becoming increasingly serious, most of the contaminated basins are located in pressure areas. In Table 1 are listed²² the known localities of sea-water intrusion in California as of 1950; the areas in acres underlain by salt water; and the type of aquifer—con-

NOTE.—This paper by Paul Baumann was published in August, 1951, as *Proceedings-Separate No. 86*. The numbering of footnotes, illustrations, tables, and equations in this Separate is a continuation of the consecutive numbering used in the original paper.

¹⁸ Lecturer in Civ. Eng., Univ. of California, Berkeley, Calif.

¹⁹ "Nota in verband met de voorgenomen put boring nabij Amsterdam" (Notes on the probable results of the proposed well drilling near Amsterdam), by W. Badon Ghyden, *Tijdschrift van het Koninkrijk, Instituut van Ingenieurs*, The Hague, 1888-1889, p. 21.

²⁰ "Die Wasserversorgung einiger Nordseebäder" (The water supply on parts of the North Sea coast), by B. Herzberg, *Journal Gasbeleuchtung und Wasserversorgung*, Vol. 44, Munich, 1901.

²¹ "Specific Gravity of Sea-Water and the Ghyben-Herzberg Ratio in Hawaii," by C. K. Wentworth, *Transactions, Am. Geophysical Union*, Part IV, 1939, pp. 690-692.

²² "Sea-Water Intrusion into Ground Water Basins Bordering the California Coast and Inland Bays," by H. O. Banks, G. B. Gleason, and R. C. Richter, Report No. 1, Water Pollution Investigations, Div. of Water Resources, State of California, 1950.

finned or unconfined. Seven of the known areas of sea-water intrusion are situated in pressure aquifers, three in non-pressure aquifers, one in a semi-confined aquifer, and two in undetermined aquifers. Of the 90,525 acres that have been found to be underlain by sea water, 87,965 acres occur in pressure areas. To control intrusion in pressure areas, methods other than spreading must be employed. Current studies in California²³ include the technical and economic feasibility of constructing artificial subsurface barriers, and also of creating fresh-water pressure ridges above mean sea level by means of injection wells.

The evaluation of the mound width developed by a circular spreading ground above an inclined impervious stratum (Eq. 124) showed that a circle of 500-ft radius provided a mound width approaching 10,000 ft. The same equation

TABLE 1.—KNOWN AREAS AND TYPES OF AQUIFERS INVADED BY SEA WATER IN CALIFORNIA, AS OF 1950

Ground-water basin or valley	Area ^a (acres)	Type of aquifer	Authority
San Diego.....	Not known	Unconfined	A. J. Ellis and C. H. Lee ^b
San Luis Rey.....	Not known	Unconfined	A. J. Ellis and C. H. Lee ^b
Santa Margarita.....	Not known	Unconfined	A. J. Ellis and C. H. Lee ^b
Central Coastal Plain.....	3,500	Confined	G. Zander and G. B. Gleason ^c
West Coastal Plain.....	16,000	Confined	G. Zander and G. B. Gleason ^c
Malibu Creek.....	160	Not known
Trancas Creek.....	Not known	Not known
Oxnard Plain.....	500 ^d	Confined	H. Conkling ^e
Salinas.....	7,000	Confined	T. R. Simpson ^f
Coastal strip ^g	530	Confined	J. M. Haley ^h
Pajaro.....	435	Confined	J. M. Haley ^h
Santa Clara.....	60,000	Confined	W. O. Clark ⁱ
Sacramento-San Joaquin ^j	2,400	Semi-confined	C. F. Tolman ^k
Total known area.....	90,525

^a Area underlain by salt water, in acres. ^b *Water Supply Paper No. 446*, U. S. Geological Survey, 1919. ^c *Bulletin No. 53*, Div. of Water Resources, State of California, 1947. ^d Approximately 500 acres. ^e *Bulletin No. 16*, Div. of Water Resources, 1933. ^f *Bulletin No. 52*, Div. of Water Resources, State of California, 1946. ^g Between Salinas Valley and Pajaro Valley. ^h "Santa Cruz-Monterey Counties Investigation," *Bulletin No. 5*, State Water Resources Board, State of California (publication pending). ⁱ *Water Supply Paper No. 519*, U. S. Geological Survey, 1924. ^j Sacramento-San Joaquin Valley between Pittsburgh and Antioch. ^k "Ground Water," by C. F. Tolman, McGraw-Hill Book Co., Inc., New York, N. Y., 1937, pp. 378-379.

may be applied to determine the mound width resulting from a recharge well. This would have application if consideration were being given to the maintenance of a fresh-water barrier along a coast in an unconfined aquifer using a line of recharge wells parallel to the coast. The mound width would determine the spacing of the wells. Since this width is independent of the spreading area size and depends only on the rate of aquifer recharge, q_0 , and the physical parameters K , a_0 , and i ; an estimate can be made based on a representative rate of well recharge. Two published accounts giving quantitative measurements of recharge wells in southern California^{24,25} show average recharge rates varying from 0.52 to 2.92 acre-ft per day. Taking a value of $q_0 = 3$ acre-ft per day as an example and using the author's values of $a_0 = 100$ ft, $K = 100$ ft per hr, and

²³ "Proposed Investigational Work for Control and Prevention of Sea-Water Intrusion into Ground Water Basins," by H. O. Banks and M. Bookman, Report to State Water Resources Board, Div. of Water Resources, State of California, 1951.

²⁴ "Spreading Water for Storage Underground," by A. T. Mitchelson and D. C. Muckel, *Technical Bulletin 573*, U. S. Dept. of Agri., Washington, D. C., 1937.

²⁵ "Surface Spreading-Operations by the Basin-Method and Tests on Underground Spreading by Means of Wells," by D. A. Lane, *Transactions*, Am. Geophysical Union, Vol. 15, 1934, pp. 523-527.

$i = 0.01$, Eq. 124 yields a mound width, W , of 54.5 ft. Actual wells would need to be spaced even closer than this distance to develop a suitable mound height and to leave no intervals between wells without fresh-water flow. In this example, the mound width would have reached only 54.0 ft at a distance of 1740 ft downstream from each well. The spacing suggests that economic considerations alone would govern in applying this method to control sea-water intrusion. Having a line of recharge wells with only some 50 ft between each well indicates that recharge by spreading would be a better method in an actual situation approximating these conditions.

MAX SUTER,²⁶ M. ASCE.—This paper is an excellent mathematical treatment of the problem of water recharge, a problem that is becoming more and more important and on which the present literature is relatively scarce. The author shows how exceedingly complicated the mathematical part of the purely hydraulic phases can become in skilled hands unafraid of hard work. In practical applications of ground-water recharge many additional factors appear, such as nonuniformity of the soil and basin boundaries, variations in the chemical and physical character of the recharge water, silting of the soil (either by silt in the recharge water or as an effect on the soil particles), pollution of ground water, influences on the temperature and chemical composition of the ground water, predictions of rate of inflow, duration of inflow and of periods between recharges, pumpage in the vicinity, and backwater effects and general variations in ground-water levels due to rainfall and floods. Many of these factors cannot be represented by mathematical formulas as they depend too much on local conditions. It is good, therefore, to have some calculating methods for those phases that are amenable to mathematical treatment and in this respect this paper marks a great step forward.

Much of the mathematics used in the paper is probably unfamiliar to many practical engineers, as few of them had or remember differential equations. It may be asked, therefore, if all this higher mathematics is necessary for the solution of the problem, and if it is justified in view of the simplified assumptions that are made. In the following, only a few examples will be discussed to show the general trend of an answer to the foregoing question. To treat every development in the paper in the same manner would fill a book.

Eq. 10a checks the writer's Fourier series expansion provided the member $\frac{1}{2} a_0$ is dropped in the general formula for the Fourier series—

$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} [a_n \cos (n x) + b_n \sin (n x)] \dots \dots \dots (127)$$

The use of the Fourier series expansion makes Eq. 10b look formidable when it actually is nothing else than

$$y = \left(\frac{q}{K a_0} - i \right) (L_d - x) (1 - e^{-\beta^2 i}) \dots \dots \dots (128)$$

To obtain this result it is not necessary to go through the Fourier series expansion.

²⁶ Head, Eng. Research, State Water Survey, Urbana, Ill.

Applying condition 1 to Eq. 9,

$$C_1 \cos \frac{(2n-1)\pi}{2L_d} x = - \left(\frac{q}{K a_o} - i \right) (L_d - x) \dots \dots \dots (129)$$

and, if this special value is introduced into Eq. 9, the result is a derivation of Eq. 128 without the detour of the Fourier series expansion. The development of Eq. 128 is based on Eq. 7 as the approximate equation of the stable mound at $t = \infty$. Referring to Fig. 1, Eq. 7 is the actual formula for y' , not y . A somewhat simpler expression than Eq. 128 is obtained by using y . Eq. 7 is replaced by

$$y = H_d - \left(\frac{H_d}{L_d} - i \right) x \dots \dots \dots (130)$$

Similarly, in the place of Eq. 14,

$$y = \left(i - \frac{H_d}{L_u} \right) x - H_d \dots \dots \dots (131)$$

If the same procedure used in the derivation of Eq. 128 is applied to Eq. 130, a replacement of Eq. 128 becomes

$$y = \frac{H_d}{L_d} (L_d - x) (1 - e^{-\beta^2 i}) \dots \dots \dots (132)$$

The principles here developed can be expanded to other formulas in the paper, and it can be seen that, without recourse to the Fourier series expansions, formulas are obtained that are simpler in form and easier for numerical calculations than those in the paper. This is important as the Fourier series, although beautiful in theory, are notoriously cumbersome for numerical calculations.

It may also be questioned if the straight-line slope is the correct approximation for the steady mound. Eq. 1 is based on the assumption of Jules Dupuit

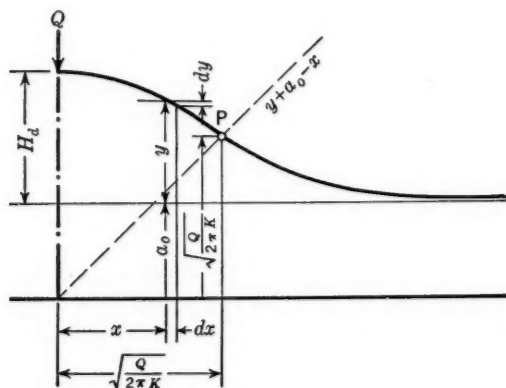


FIG. 22

that the velocity in a vertical is constant and horizontal and in an amount equal to that defined by the Darcy law—namely, $v = K (dy/ds)$ which is replaced by $v = K (dy/dx)$ for flat slopes for the sake of integrality of the equations.

The Dupuit assumptions are nonpotential but give close approximation in flat slopes, where the sine of the slope can be replaced by the tangent. The following developments are based on these

assumptions and show that even these simplifications furnish curves of the general character obtained by the potential theory and by Mr. Baumann's experiments. Assuming a point source with circular runoff around it, the areas are as follows:

At distance	Area
x	$2 \pi x (y + a_o)$
$x + dx$	$2 \pi (x + dx) (y + a_o - dy)$

Therefore, the change in area is

$$dA = 2 \pi [(a_o + y) dx - x dy] \dots \dots \dots (133)$$

—neglecting the higher differential $dx dy$ (see Fig. 22).

Inasmuch as

$$v = \frac{Q}{A} = K \frac{dy}{dx} \dots \dots \dots (134)$$

TABLE 2.—CHARACTERISTICS OF THREE CASES OF WATER SURFACE CURVES

Case	Value of dA	Area	Velocity	Slope	Water surface
1	0	Constant	Constant	Constant	Straight line
2	Positive	Increasing	Decreasing	Flattening	Concave line
3	Negative	Decreasing	Increasing	Steepening	Concave line

there are three cases for the water surface curve (see Table 2). When do these cases occur? Since

$$dA = 0 = (a_o + y) dx - x dy \dots \dots \dots (135)$$

$$\frac{dx}{x} = \frac{dy}{a_o + y} \dots \dots \dots (136)$$

Integrated,

$$\log x + \log C = \log (a_o + y) \dots \dots \dots (137)$$

and

$$a_o + y = C x \dots \dots \dots (138)$$

in which, at point P:

$$C = \frac{dy}{dx} \dots \dots \dots (139)$$

For flat slopes $C < 1$ but the actual value is not yet known. Under the conditions assumed in the paper

$$C = \frac{H_d}{L_d} \dots \dots \dots (140)$$

Since—

$$Q = 2 \pi K x (a_o + y) \frac{dy}{dx} \dots \dots \dots (141)$$

— $dA = 0$ at a point x, y defined as

$$y = \sqrt{\frac{Q}{2 \pi K}} - a_o \dots \dots \dots (142a)$$

and

$$x = C \sqrt{\frac{Q}{2 \pi K}} \dots \dots \dots (142b)$$

The condition dA exists therefore only in the immediate vicinity of an elevation y fixed by natural conditions. The value of C at that point can be determined by a study of the curve of the water surface. Integrating Eq. 141, an equation of the steady mound is derived:

$$y^2 + 2 a_o y - \frac{Q}{\pi K} \log x + C = 0 \dots \dots \dots (143)$$

For an infinitely large spreading ground, $C = \infty$ and for a limited spreading ground $C = \frac{Q}{\pi K} \log L_d$. In these two cases, Eq. 143 yields

$$y = -a_o \pm \sqrt{a_o^2 + \frac{Q}{\pi K} \log x - C} \dots \dots \dots (144a)$$

and

$$y = -a_o \pm \sqrt{a_o^2 + \frac{Q}{\pi K} \log \frac{x}{L}} \dots \dots \dots (144b)$$

—both of which lead to

$$\frac{dy}{dx} = -\frac{Q}{2\pi K (a_o + y) x} \dots \dots \dots (145)$$

Eq. 145 becomes zero only for $x = \infty$, since $y_{\max} = H_d$. On second derivation,

$$\frac{d^2y}{dx^2} = \frac{Q}{2\pi K} \times \frac{1}{x^2} \times \frac{1}{a_o + y} \left(1 - \frac{Q}{2\pi K} \times \frac{1}{(a_o + y)^2} \right) \dots \dots \dots (146)$$

Eq. 146 becomes zero for $x = \infty$ and for $y = \sqrt{\frac{Q}{2\pi K}} - a_o$.

The point with $dA = 0$ is therefore also a point of inflection and its x -coordinate, from the condition defined by Eq. 133, is computed by

$$x = \sqrt{\frac{Q}{2\pi K}} \dots \dots \dots (147)$$

Therefore, this point lies on the line $a_o + y = x$, and in Eq. 138

$$C = \frac{dy}{dx} = -1 \dots \dots \dots (148)$$

This steep slope explains why, in actual field and laboratory observations, there is a sharp drop near the recharge area.

A slope of 45° is never observed because, in the field and in the model, there are influences that flatten the slope—namely:

- (a) For steep slopes Dupuit's assumptions do not yield a good approximation; and
- (b) There is an area of inflow instead of a point of recharge. Eq. 146 shows two additional properties of the surface configuration:

$$(1) \frac{d^2y}{dx^2} \text{ is positive if } y > \sqrt{\frac{Q}{2\pi K}} - a_o$$

such a curve is convex between the source of recharge and point P. On the other hand,

$$(2) \frac{d^2y}{dx^2} \text{ is negative for } y < \sqrt{\frac{Q}{2\pi K}} - a_0$$

and such a curve is concave from point P outward. Considering these properties of the curve of the steady mound it does not seem appropriate to apply a refined computation involving an assumption that the water surface can be represented by a straight line.

The author presents curves of the type here developed, but also inverse curves. However, it is the writer's belief that the drop shown near the outflow is due to a backwater of less height than the critical depth²⁷:

$$h_c = \frac{2}{\pi} \times \frac{Q_{\max}}{K} \dots \dots \dots (149)$$

D. P. KRYNINE,²⁸ M. ASCE.—The history of engineering reveals a great number of cases when successful structures were built and engineering operations performed from experience and partly by intuition, but without adequate theoretical background on the part of engineers. Theoretical clarifications based on physics and mechanics were given generally much later by scientists and scientifically inclined engineers other than the original ones. Such, for instance, was the case of ancient earth dams that were built since times immemorial without their builders having any notion of soil mechanics or subsurface flow net theories. Conversely, in the case of the present paper we have an extraordinary example of a modern engineer who for many years was engaged in the technical direction of spreading operations on a large scale in Southern California and then himself produced a complete theory of spreading tending to explain the minute details of the phenomenon. Under given circumstances it is exceedingly difficult to discuss this interesting paper, not only because of the abundant and rather complicated mathematical analysis presented, but chiefly because of the fact that the author, by virtue of his practice, is strongly prepared to defend his views. The latter, in addition, are backed by well-performed experiments which the writer had an opportunity to examine personally in 1948. The writer, therefore, wishes to limit his discussion mostly to the comparison of the author's philosophy of approach to the given problem with his own views.

After having read Mr. Baumann's paper the writer became convinced that at least three problems of the underground flow belong to the same class. These are—the theory of consolidation; drainage of the pavement base course;²⁹ and spreading. From these three the spreading problem is the most complicated because in this case the flow is confused especially at the experimental stage, by capillarity, mostly above the saturation line, and also by the boundary conditions, as at the bottom of the experimental box. In fact, the latter conditions are indicated by the travel fringes marked 5, 6, and so on, minutes in

²⁷ "Technische Hydraulik," by Charles Jaeger, Birkhäuser, Basel, Switzerland, 1949, p. 366.

²⁸ Cons. Engr., San Francisco, Calif.

²⁹ "Base Course Drainage for Airport Pavements," by A. Casagrande and W. L. Shannon, *Proceedings-Separate No. 75*, ASCE, June, 1951.

Figs. 19 and 20. In the opinion of the writer these fringes are produced by the colored liquid that falls to the bottom of the box through water and spreads horizontally due to the combined action of hydraulic head and momentum. The writer observed a somewhat similar phenomenon in his experiments on horizontal capillarity in cylindrical tubes filled with dry sand. In the latter case water just under a small head first very quickly spreads horizontally along the bottom of the tube and afterwards invades the tube in a capillary way moving upward.³⁰ The writer believes that the use of a parabolic bottom in the experimental box (Fig. 21) was a fruitful idea since in such a way the true character of flow in nature is better duplicated by the experiment than in the case of a horizontal bottom of the experimental box.

The solution of the three aforementioned problems is based in one way or another on the well-known differential equation of heat transfer through a prismatic, nonradiating bar with a constant coefficient proportional to the coefficient of permeability k . In the case of subsurface flow the coefficient k is not constant, however, and gradually decreases as either the process of drainage or spreading advances. This is a pronounced phenomenon in the case of soils with fine-grained admixtures. The situation is better in the case of flow through coarse pervious materials since in this case a relative constancy of the coefficient of permeability during drainage or spreading may be expected. Hence, the author's statement that " * * * spreading grounds are located in soils that approach idealized conditions extending over great distances" is encouraging. It is to be noticed that the same favorable condition prevails in the case of runway bases made of gravel or similar materials.

The hydraulics of the open-channel flow had existed before the advent of the theories of the subsurface flow; and undoubtedly the latter have been influenced by the former. Thus, in the theories of quasi-horizontal subsurface flow the existence of a horizontal or slightly inclined impervious bottom (like those occurring in some rivers) is predicated. In reality, however, the earth mass may be pervious to an indefinite depth and saturated with ground water. The most realistic explanation why an impervious bottom is introduced in the theories of quasi-horizontal subsurface flow is the necessity to have a plane from which the ordinates of the subsurface stream could be measured; otherwise the formulas cannot be derived. Such is the case in the Dupuit formula, the theory of the pumping test, the theory of the drainage of a pavement base,²⁹ and, finally, the theory of spreading as presented in this paper.

The writer feels, however, that, in the case of a very deep ground-water stream with large values of a_0 , spreading would affect the lower part of this stream very little, if at all. It is stated sometimes in soil mechanics that that water has no shearing strength, but this statement is of a relative value only. The very concept of viscosity is based on the possibility that shearing stresses exist in a viscous fluid such as water. The hydraulic gradient pushing the spreading stream down is larger than that of the underlying ground water; hence, the spreading stream moves with larger velocities than the underlying ground water. This causes shearing stresses and a corresponding increase of

³⁰ "Some Capillary Phenomena in Sandy Materials," by D. P. Krynine, *Proceedings*, 29th Annual Meeting, Highway Research Board, Washington, D. C., 1950, pp. 520-530.

velocities in the ground water, which fact permits gradual incorporation of the mound wave into the original ground-water stream. The shearing stresses have to decrease rapidly from the surface of the original ground water downward and at a certain depth become negligible for all practical purposes. This pattern may be easily visualized by analogy from the diagram of shearing stresses in a body with a horizontal or slightly inclined upper surface over which a heavy object with a plane bottom surface is sliding. The writer's reasoning is valid, in his opinion, at least for the first and second phases of the spreading process.

In conclusion the writer wishes to congratulate Mr. Baumann on his assiduous and conscientious work which represents a valuable contribution to the mathematical theory of spreading.

PAUL BAUMANN,³¹ M. ASCE.—In preparing this paper over a period of several years, the writer had two principal objectives, namely—to find sound, theoretical interpretation of observed ground-water phenomena related to spreading, and to clarify attendant, fundamental concepts which appeared to be afflicted, to some degree at least, by confusion. Above all, it was felt that the wave phenomenon and the significance of controls had not been fully evaluated and recognized, respectively.

Mr. Todd seems to interpret the writer's expression, "to at least mean sea level," as falling short of possibly 3.5 ft above mean sea level. Such was not the intention. Since 1950 the writer has been in charge of an investigation, by the Los Angeles County Flood Control District, of economic means for the stemming of sea-water intrusion in the West Coastal Basin. This basin (which Mr. Todd refers to in Table 1 as "West Coastal Plain") has a shore line about 12 miles in length along which about two thirds of the principal aquifer, known as Silverado Zone, are confined and the balance unconfined. The latter is predominantly located in the southern part of the area. In 1950 spreading tests were conducted to confirm the unconfined character of the aquifer there, to determine the permeability of the soil, and to ascertain the salinity of the ground water. The investigation led to the conclusion that, in this area, sea-water intrusion could best be controlled by means of a liquid barrier, created through spreading from a strip area or from shallow gravel-filled injection pits, 50 ft apart. This barrier would have to be located well inland from the seashore so as to have the assurance of control. Therein lies a distinct difference between the mechanics of a liquid barrier in an open and a confined aquifer. The latter, as Mr. Todd correctly states, is not treated in the paper, although a recharge test was conducted that same year on an abandoned well in the confined zone of the aquifer. Both tests plus a third one in the northern quasi-unconfined area were described in a report by the Los Angeles County Flood Control District, prepared in the spring of 1951, together with a study on the location and components of a complete liquid barrier over the entire 12-mile length. Finally, it included a recommendation for a pilot test on recharge wells, extending over a 1-mile front in the confined area, some 2,000 ft inland from the seashore. Although the chloride concentration at this location may

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correspond to a coefficient $\alpha_s = 1.01$, and may call for a rise of the barrier above mean sea level of 2 ft \pm , the chloride concentration at the proposed injection pits of the open aquifer is very much smaller, so as to require a rise there of only a few tenths of a foot above mean sea level. Hence, the wording of the paper was meant to imply that, whereas in some locations the rise of the mound above mean sea level would be negligible to stem intrusion, it could be appreciable in other locations. Where the latter happens to be the case, each foot of extra rise may indeed represent a considerable part of the total effort. Hence, Mr. Todd's point is well taken.

Fortunately, the conditions in the open area of the West Coastal Basin are not as difficult as they would be if the physical parameters used in the writer's prototype example applied. These values actually apply to a spreading ground in a debris cone area and not to the open aquifer of the West Coastal Basin. The respective values would be more nearly: $q_o = 0.25$ cu ft per sec = 900 cu ft per hr; $a_o = 300$ ft; $K = 10$ ft per hr; and $i = 0.004$. Hence, the corresponding mound width, $W = 75$ ft, would assure complete closure based on the aforementioned pit spacing of 50 ft.

In reading Mr. Suter's able discussion, one should bear in mind two fundamental premises: (1) The writer confined linearization of the stable or stationary mound to the two-dimensional case only and (2) "the improbability of the existence of a circular control" was emphasized in connection with the three-dimensional case. The linearization of the two-dimensional mound is a permissible approximation as long as $y \ll a_o$ in accordance with the basic premise. The discrepancy between the linearized and the true, stable mound shown in Fig. 14 is exaggerated because in the comprehensive model tests the aforementioned, basic premise was not satisfied. In fact, the height of the stable mound was nearly one half of the initial ground-water depth. This exaggeration was intentional to facilitate observation on the one hand and to ascertain the effect of this deviation from the basic premise on the theoretical results. It is a well-known fact that mathematics often furnishes valid results well beyond the imposed limitations.

Mr. Suter makes a commendable effort at simplification of Eq. 10b by introducing boundary condition 1 in Eq. 9. Apparently the statement, "Eq. 9 still does not satisfy condition 1 * * *" was thereby overlooked. Naturally, if Eq. 9 does not satisfy condition 1, then condition 1 does not satisfy Eq. 9. Hence, it is necessary, first of all, to transform Eq. 9 into an equation which does satisfy condition 1. This was accomplished through Eq. 10b. It is regrettable, of course, that condition 1 is singularly difficult to satisfy and that the elegant operation suggested by Mr. Suter does not answer the purpose. Unfortunately, it leads to the result expressed through Eq. 128, which describes a family of straight lines, radiating from the control toward the spreading ground. Hence, in accordance therewith, not only are the undisturbed ground-water surface and the surface of the stable mound linear but also all the intermediate surface configurations of the growing mound. Again, referring to Fig. 14, it is obvious that such is not the case. All equations derived from Eq. 4 must necessarily satisfy the latter. This is true of all of the writer's equations for the growing and the disappearing mound, but it is not true of Eq. 128.

Eqs. 16 and 17 which express the entire, two-dimensional phenomenon are not as formidable as they may appear at first glance. For $i = 0$, $L_u = L_d$, and Eqs. 17 check Eq. 10b. Length, L_d , may be any finite distance. Hence, in the case of L_d considerably in excess of the one represented in Fig. 2, it is possible to determine the progress and shape of the wave in both the upstream and downstream direction for such arbitrary time elements as correspond to the first phase. This is shown in Fig. 23 which is based on the same numerical values as the dash lines in Fig. 2 except for $L_d = 15$ ft instead of 4 ft. It shows that in this case the time required for the wave to reach the control, and for discharge

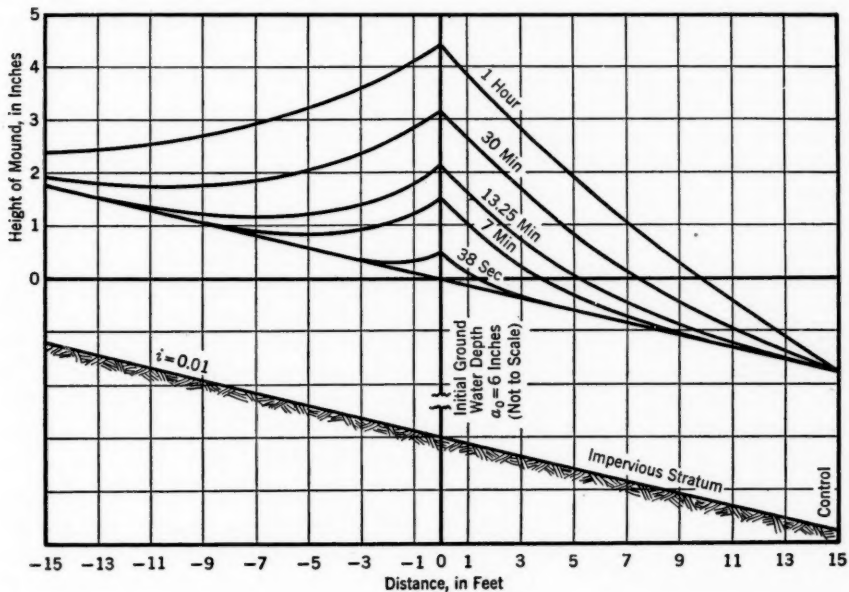


FIG. 23

in addition to the initial ground-water flow to commence, is of the order of 13 min as compared to 0.5 min in the first case. It also shows the flexibility of equations of this type. For $L_u = L_d$ ($i = 0$) Eqs. 17 also check the analogue of the equation³² for heat flow in a nonradiating, prismatic bar of semi-infinite length which reads (in terms of ground-water parameters):

$$y = \frac{q_0 x}{K a_0 \sqrt{\pi}} \left\{ \frac{e^{-x^2 j^{-1/4}}}{\sqrt{\frac{x^2}{4j}}} - \sqrt{\pi} \left[1 - \Phi_{(x)} \left(\frac{x^2}{4j} \right) \right] \right\} \dots \dots \dots (150)$$

in which (see Eq. 5) $j = \frac{K a_0 t}{\mu}$; and $\Phi_{(x)} = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$, the probability

³² "Heat Conduction," by Ingersoll, Zobel, and Ingersoll, McGraw-Hill Book Co., Inc., New York, N. Y., 1948, p. 110.

integral which is readily determined from tables.³³ For $x = 0$, Eq. 150 reduces to

$$y = q_0 \sqrt{\frac{4t}{K a_0 \pi \mu}} \dots \dots \dots (151)$$

which also checks the corresponding term derived from Eqs. 17.

Mr. Suter states correctly that the writer's equations for y are actually those for y' according to Fig. 1, and that the relations as shown in Eqs. 130 and 131 should be used for the respective transformation. Unfortunately, however, this transformation does not free Eq. 132 of the discrepancy attending Eq. 128.

In the light of the foregoing, Mr. Suter's check on the validity of the writer's linearization does not seem pertinent in that his development of Eqs. 133 to and including 148, is entirely based on three-dimensional flow for $i = 0$. Actually the writer's formulas in Part II, Section 1, correspond to the aforementioned ones, except that Mr. Suter assumes stationary extracted, flow (positive right side in Eq. 141) and therefore a circular control, whereas the writer's development is based on unsteady injected flow (negative right side, in Eq. 107) and no control. Furthermore, Mr. Suter assumes a point source of "inflow" (Fig. 22) instead of a circular spreading ground as used by the writer (Fig. 9). It is at once apparent that reduction of the radius of the spreading ground such as to approach a point source would result in a surface configuration quite different from that shown in Fig. 22.

In setting up the equations for the two cylindrical areas a distance of dx apart, preceding Eq. 133, Mr. Suter introduces h negative increment dy for a positive increment dx , whereas both should be entered positive. It is up to mathematics to tell in the end whether dy is positive or negative.

Hence, Eq. 138 for the condition $dA = 0$ should read

$$x(a_0 + y) = C \dots \dots \dots (152)$$

which defines a family of square hyperbolas with the y -axis and a line parallel to, and a distance a_0 below, the x -axis as asymptotes. The intersection of any of these hyperbolas with the undisturbed ground-water surface corresponds to the distance L_d at which the circular control is assumed. This solution satisfies the fundamental law that it is impossible to maintain all three—area, velocity, and slope constant—for increasing x . These hyperbolas are a special case of the so-called "cone of expression." It is important to emphasize, however, that a recharge well cannot be treated as a water well "in reverse"—that is, by assuming stationary flow as Mr. Suter did. The reason is that a water well is in itself a control whereas the recharge well is not. Hence, in the latter case the flow is unsteady and radiates from the well uniformly to any distance in form of a wave without ever becoming stationary within finite limitations. In the course of this performance the rate of recharge must diminish as illustrated in Fig. 10 (dash line). This diminution in the rate of recharge is, purely and simply, a hydromechanical necessity; yet, it has been attributed to causes

³³ "Heat Conduction," by Ingersoll, Zobel, and Ingersoll, McGraw-Hill Book Co., Inc., New York, N. Y., 1948, see appendix.

other than the real one, by recent investigators. The writer is by no means oblivious to the fact that causes of an entirely different nature exist; but he believes in taking the first step first and in taking the probable into consideration ahead of the possible.

In Mr. Suter's final paragraph, reference is made to the critical depth at the control in accordance with Charles Jaeger's determination,²⁷ which compares with Eq. 102. Both equations are approximations. Contrary to the writer's development in Section 4, Part I, Mr. Jaeger arrives at his value through substitution of circular arcs for the parabolic equipotential lines. This leads to a sharp drawdown of the phreatic line and a vertical tangent at the control. The point of contact of this tangent determines the critical depth. In either case the critical depth is defined as the minimum height of the phreatic line at the control regardless of how far below it the backwater surface may be. The true value of h_c probably lies between that defined by Eqs. 149 and 102.

Mr. Krynine's reference to a very deep ground-water stream reveals a keen concept of the mathematical premises necessary to make solution possible. The depth to the impervious stratum and its slope may never have been explored. In fact, a clear-cut, impervious stratum may not even exist. Instead, the aquifer may terminate in a merged zone which, under the influence of the critical surcharge, has broken down into a quasi-impervious soil. An effective, impervious stratum must then be assumed, preferably in the form of a horizontal plane. It must not be overlooked, however, that the pressure conditions, even in a deep ground-water stream, in an open aquifer, may be decisively affected through the creation of a spreading mound of commensurate height. As shown in Figs. 2 and 23, such a stream, regardless of how mighty, may be checked by strip spreading. The original ground-water flow downstream from the spreading ground is then replaced by infiltrated water from the latter during the first, and part of the second, phase. It is axiomatic that to accomplish this replacement the infiltrated flow per unit length of spreading ground must be greater than the corresponding initial ground-water flow, which reveals the fact that the spreading ground itself is a control even if the mound never rises to the surface. Hence, spreading grounds are powerful hydraulic "weapons" and must be operated with proper appreciation of that fact.

The progressive transmission of motion of the spreading wave from its surface to the initial ground-water stream at any depth through the medium of shearing stresses is a telling example of Mr. Krynine's insight into these phenomena. His reference to the fruitfulness of the parabolic bottom used in the model is no doubt closely related to the significance of shearing stresses. In this connection it is well to realize, however, that the mathematical treatment, based on the Boussinesq equation in general and on Eq. 4 (heat flow analogue) in particular is afflicted with a physical inconsistency. This is due to the neglect of inertia forces. It implies instantaneous instead of progressive acceleration through the spreading mound of initial ground-water bodies and instantaneous transmission of pressure to the control or infinity, during the first phase. The introduction of the intuitive, unique flow function in Part I, Section 3, tends to mitigate the aforementioned inconsistency in that a finite

wave length is associated with a finite time element. It is interesting to note from Fig. 14 that equations based on the latter concept led to closer agreement with the test results than those based on Eq. 4.

In conclusion, the writer wishes to express his sincere appreciation to the discussers of the time and effort which were necessary to compose their valuable contributions to this paper. In the light of the combined efforts it is hoped that ground-water phenomena related to spreading will be more readily understood in the future than they have been in the past.